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Multiple Mellin Transform involving I-functions of Several Variables

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Abstract:
In this paper, we derive an explicit formula for the multidimensional Mellin transform involving the product of algebraic radical functions and the multivariable I- function. The main result is obtained in closed form as a higher-order multivariable I- function by utilizing a classical Mellin-type integral formula due to Marichev O I, along with a discussion of relevant corollaries.

Key Words: Multivariable I- functions, Gamma function, Pochhammer symbol, Multidimensional Mellin transform.

1. INTRODUCTION

The generalized Fox's *H*- function, namely *I*- function of *r*- variables introduced by Prathima, Nambisan and Shantha Kumari [8, p.38] is defined and represented as:

$$I[z_1,...,z_r]=I^{0,n:\ m_1,n_1;...;m_r,n_r}_{p,q:\ p_1,q_1;...;p_r,q_r}\left[\begin{array}{l|l} z_1 & {}_1(a_j;\alpha_j^{(1)},...,\alpha_j^{(r)};A_j)_P : {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};...;{}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ \vdots & \\ z_r & {}_1(b_j;\beta_j^{(1)},...,\beta_j^{(r)};B_j)_Q\ : {}_1(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};...;{}_1(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{array}\right]$$

$$=\frac{1}{(2\pi\,\omega)^r}\int_{L_1}...\int_{Lr}\theta_1(s_1)...\theta_r(s_r)\ \phi(s_1,...,s_r)\ z_1^{s_1}...z_r^{s_r}ds_1...ds_r, \qquad (1.1)$$

where $\phi(s_1,...,s_r)$ and $\theta_i(s_i),\ i=1,2,...,r$ are given by,

$$\phi(s_1,...,s_r)=\frac{\prod_{j=1}^{n}\Gamma^{A_j}(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_i)}{\prod_{j=1}^{q}\Gamma^{B_j}(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i)\prod_{j=n+1}^{p}\Gamma^{A_j}(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_i)}, \tag{1.2}$$

$$\theta_i(s_i)=\frac{\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}(d_j^{(i)}-\delta_j^{(i)}s_i)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}(c_j^{(i)}-\gamma_j^{(i)}s_i)}. \tag{1.3}$$

Also $z_i\neq 0\ (i=1,...,r)$, $\omega=\sqrt{-1}$, $m_j,n_j,p_j,q_j\ (j=1,...,r)$, n , p , q are non-negative integers such that $0\le n\le p$, $q\ge 0$, $0\le m_j\le q_j$, $0\le n_j\le p_j$ $(j=1,2....,r)$ (not all zero simultaneously). $\alpha_j^{(i)}$ $(j=1,2,...,p\ ,\ i=1,2...,r)$, $\beta_j^{(i)}$ $(j=1,2,...,q,\ \ i=1,2,...,r)$, $\gamma_j^{(i)}$ $(j=1,2,...,p_i,\ i=1,2,...,r)$, and $\delta_j^{(i)}$ $(j=1,2,...,q_i,\ i=1,2,...,r)$ are positive numbers. $a_j\ (j=1,2,...,p)$, $b_j\ (j=1,2,...,q)$, $c_j^{(i)}$ $(j=1,2,...,p_i,\ i=1,2,...,r)$, and $d_j^{(i)}$ $(j=1,2,...,q_i,\ i=1,2,...,r)$ are complex numbers. The exponents $A_j\ (j=1,2,...,p)$, $B_j\ (j=1,2,...,q)$, $C_j^{(i)}\ (j=1,2,...,p_i,\ i=1,2,...,r)$, and $D_j^{(i)}$ $(j=1,2,...,q_i,\ i=1,2,...,r)$ of various gamma functions may take non integer values. The I- function of r-variables is analytic if

$$\Psi_i=\sum_{j=1}^{p}A_j\alpha_j^{(i)}-\sum_{j=1}^{q}B_j\beta_j^{(i)}+\sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)}\ \le 0, i=1,2,...,r.$$

The integral (1.1) converges absolutely if $\left|\arg(z_i)\right|<\frac{1}{2}\Delta_i\pi$, $i=1,2,...,r$ where

$$\Delta_i=-\sum_{j=n+1}^{p}A_j\alpha_j^{(i)}-\sum_{j=1}^{q}B_j\beta_j^{(i)}+\sum_{j=1}^{m_i}D_j^{(i)}\delta_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}\delta_j^{(i)}$$

$$+\sum_{j=1}^{n_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\gamma_j^{i}>0 \tag{1.4}$$

Remark: 1

If $D_j^{(i)}=1\ (j=1,2,...m_i,\ \ i=1,2,...r)$ in (1.1), then I - function will be denoted by:

$$\overline{I}[z_1,.....,z_r]=I_{p,q:\ p_1,q_1;...;p_r,q_r}^{0,n:\ m_1,n_1;...;m_r,n_r}\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2 \end{array}\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}...\int_{L_r}\overline{\theta}_1(s_1)\ ...\ \overline{\theta}_r(s_r)\phi(s_1,...,s_r)\ z_1^{s_1}...\ z_r^{s_r}\ ds_1...\ ds_r \tag{1.5}$$

where

$$I_1 = {}_1(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)};A_j)_p : {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\ldots;{}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r},$$

$$I_2 = {}_1(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)};B_j)_q : {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1}, {}_{m_1+1}(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};\ldots;{}_1(d_j^{(r)},\delta_j^{(r)};1)_{m_r}, {}_{m_r+1}(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r},$$

$$\overline{\theta_i}(s_i)\frac{\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-\delta_j^{(i)}s_i)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}(c_j^{(i)}-\gamma_j^{(i)}s_i)}, \quad i=1,2,\ldots,r \tag{1.6}$$

and $\phi(s_1,\ldots,s_r)$ is given by (1.2).

The integral (1.5) converges absolutely if $\left|\arg(z_i)\right| < \frac{1}{2}\Delta_i'\,\pi,\ i=1,2,\ldots,r.$

where,

$$\Delta_i' = \left(-\sum_{j=n+1}^{p}A_j\,\alpha_j^{(i)} - \sum_{j=1}^{q}B_j\,\beta_j^{(i)} + \sum_{j=1}^{m_i}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i}D_j^{(i)}\,\delta_j^{(i)} + \sum_{j=1}^{n_i}C_j^{(i)}\,\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i}C_j^{(i)}\,\gamma_j^{(i)}\right) > 0,\ i=1,2,\ldots,r. \tag{1.7}$$

Remark: 2

If $C_j^{(i)}=1$ $(j=1,2,\ldots n_i)$, $D_j^{(i)}=1$ $(j=1,2,\ldots m_i)$ for $i=1,2,\ldots r$ and if $n=0$ in (1.1), then the corresponding function will be denoted by:

$$\overline{I_1}[z_1,\ldots\ldots,z_r] = I_{p,q:\,p_1,q_1;\ldots;p_r,q_r}^{0,0:\,m_1,n_1;\ldots;m_r,n_r}\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2 \end{array}\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\int_{L_r}\overline{\theta_1}^{*}(s_1)\ldots\overline{\theta_r}^{*}(s_r)\phi_1(s_1,\ldots,s_r)\,z_1^{s_1}\ldots z_r^{s_r}\,ds_1\ldots ds_r \tag{1.8}$$

where

$$I_1 = {}_1(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)};A_j)_p : {}_1(c_j^{(1)},\gamma_j^{(1)};1)_{n_1}, {}_{n_1+1}(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\ldots;{}_1(c_j^{(1)},\gamma_j^{(1)};1)_{n_r}, {}_{n_r+1}(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r}, \tag{1.9}$$

$$I_2 = {}_1(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)};B_j)_q : {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1}, {}_{m_1+1}(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};\ldots;{}_1(d_j^{(r)},\delta_j^{(r)};1)_{m_r}, {}_{m_r+1}(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r}, \tag{1.10}$$

$$\phi_1(s_1,\ldots,s_r) = \frac{1}{\prod_{j=1}^{q}\Gamma^{B_j}(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i)\prod_{j=1}^{p}\Gamma^{A_j}(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_i)}, \tag{1.11}$$

$$\overline{\theta_i}^{*}(s_i)\frac{\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-\delta_j^{(i)}s_i)\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}(c_j^{(i)}-\gamma_j^{(i)}s_i)}, \quad i=1,2,\ldots,r \ . \tag{1.12}$$

The integral (1.8) converges absolutely if $\left|\arg(z_i)\right| < \frac{1}{2}\Delta_i''\,\pi,\ i=1,2,\ldots,r.$

where,

$$\Delta_i'' = \left(-\sum_{j=1}^{p} A_j\,\alpha_j^{(i)} - \sum_{j=1}^{q} B_j\,\beta_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\,\delta_j^{(i)} + \sum_{j=1}^{n_i} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\,\gamma_j^{(i)} \right) > 0,\ i=1,2,\ldots,r. \tag{1.13}$$

For further details refer [8].

Definition: (The Mellin transform of r variables. Erdelyi et al. [2])

The Mellin transform of $f(x_1,\ldots,x_r)$ is defined as:

$$\mathrm{M}\left[f(x_1,\ldots,x_r)\ ; s_1,\ldots,s_r \right] = G(s_1,\ldots,s_r) = \int_0^\infty \ldots \int_0^\infty x_1^{s_1-1}\ldots x_r^{s_r-1} f(x_1,\ldots,x_r)\,dx_1\ldots dx_r \tag{1.14}$$

provided that the multiple integrals occurring in the right-hand side of (1.14) is absolutely convergent and $x_j > 0$, for $j=1,2,\ldots,r$; $s_j = \sigma_j + i\omega_j$ are complex parameters within a fundamental strip of convergence.

Remark: By employing the properties of the Mellin transform, Marichev O. I [4, p.133] derived the following formula:

$$\int_0^\infty x^{\rho-1}\left(1+\sqrt{1+x}\right)^{\nu}(1+x)^{-1/2}\,dx = \frac{1}{\sqrt{\pi}}\,\Gamma\left[\begin{matrix}\rho, & 1-\frac{\nu}{2}-\rho, & \frac{1-\nu}{2}-\rho \\ & 1-\nu-\rho & \end{matrix}\right] \tag{1.15}$$

provided $0 < \mathrm{Re}(\rho) - \frac{1}{2}\left(1-\mathrm{Re}(\nu)\right)$

2. Main Result:

Theorem (2.1): Let M denote the multi-dimensional Mellin transform operator of r variables given in (1.14). Let $\sigma_i\,(i=1,2,\ldots,r)$ be positive real numbers and $\nu_i, \rho_i\,(i=1,2,\ldots,r)$ be complex parameters satisfying $\mathrm{Re}(\nu_i) < 1$. Then the multiple Mellin transform involving the I - function of several variables exists and is given by:

$$\mathrm{M}\left[\left(1+\sqrt{1+x_1}\right)^{\nu_1}\ldots\left(1+\sqrt{1+x_r}\right)^{\nu_r}(1+x_1)^{-1/2}\ldots(1+x_r)^{-1/2}\,\overline{I}_1\left[z_1 x_1^{\sigma_1},\ldots,z_r x_r^{\sigma_r}\right];\rho_1,\ldots,\rho_r \right]$$

$$= \frac{1}{\left(\sqrt{\pi}\right)^r}\,\overline{I}_{1\ P,Q:\ p_1+2,q_1+2\ ;\ldots;\ p_r+2,q_r+2}^{0,0:\ m_1+2,n_1+1\ ;\ldots;\ m_r+2,n_r+1}\left[\begin{matrix} z_1 & | & I_1 \\ \vdots & & \\ z_r & | & I_2 \end{matrix}\right] \tag{2.1}$$

where

$$I_1 = {}_1\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)};A_j\right)_P : \left(1-\rho_1,\sigma_1;1\right),{}_1\left(c_j^{(1)},\gamma_j^{(1)};1\right)_{n_1},{}_{n_1+1}\left(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)}\right)_{p_1},\left(1-\nu_1-\rho_1,\sigma_1;1\right);\ldots$$

$$\ldots;\left(1-\rho_r,\sigma_r;1\right),{}_1\left(c_j^{(r)},\gamma_j^{(r)};1\right)_{n_r},{}_{n_r+1}\left(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)}\right)_{p_r},\left(1-\nu_r-\rho_r,\sigma_r;1\right) \tag{2.2}$$

$$I_2 = {}_1\left(b_j;\beta_j^{(1)},...,\beta_j^{(r)};B_j\right)_Q:\left(1-\frac{v_1}{2}-\rho_1,\sigma_1;1\right),\left(\frac{1-v_1}{2}-\rho_1,\sigma_1;1\right),{}_1\left(d_j^{(1)},\delta_j^{(1)};1\right)_{m_1},{}_{m_1+1}\left(d_j^{(1)},\delta_j^{(1)};D_j^{(1)}\right)_{q_1};...$$

$$...;\left(1-\frac{v_r}{2}-\rho_r,\sigma_r;1\right),\left(\frac{1-v_r}{2}-\rho_r,\sigma_r;1\right),{}_1\left(d_j^{(r)},\delta_j^{(r)};1\right)_{m_r},{}_{m_r+1}\left(d_j^{(r)},\delta_j^{(r)};D_j^{(r)}\right)_{q_r} \quad (2.3)$$

provided by the following conditions are satisfied:

(i) For all s_i lying in the complex contour L_i, the parameters satisfy

$$0<\mathrm{Re}(\rho_i+\sigma_i s_i)<\frac{1}{2}\left(1-\mathrm{Re}(v_i)\right). \quad (2.4)$$

(ii) $$\mathrm{Re}\left(\rho_i+\sigma_i\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>0,\ \ j=1,...,m_i;\ i=1,...,r \quad (2.5)$$

(iii) $$\mathrm{Re}\left(1-\frac{v_i}{2}-\rho_i-\sigma_i\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)>0,\ j=1,...,m_i;\ i=1,...,r \quad (2.6)$$

(iv) $$\mathrm{Re}\left(\frac{1}{2}-\frac{v_i}{2}-\rho_i-\sigma_i\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)>0,\ j=1,...,m_i;\ i=1,...,r \quad (2.7)$$

(v) $\Delta_i''>0$; $|\arg z_i|<\frac{1}{2}\Delta_i''\pi$, $i=1,2,...,r$ where Δ_i'' is given by (1.14).

Proof:

Applying (1.14) to the left-hand side of (2.1), it becomes:

$$\int_0^\infty...\int_0^\infty x_1^{\rho_1-1}...x_r^{\rho_r-1}\left(1+\sqrt{1+x_1}\right)^{v_1}...\left(1+\sqrt{1+x_r}\right)^{v_r}(1+x_1)^{-1/2}...(1+x_r)^{-1/2}\,\overline{I}_1\left[z_1x_1^{\sigma_1},...,z_rx_r^{\sigma_r}\right]dx_1...dx_r \quad (2.8)$$

Express the I - function of r variables as a contour integral (1.8), (2.8) reduces to:

$$\int_0^\infty...\int_0^\infty\Bigg[x_1^{\rho_1-1}...x_r^{\rho_r-1}\left(1+\sqrt{1+x_1}\right)^{v_1}...\left(1+\sqrt{1+x_r}\right)^{v_r}(1+x_1)^{-1/2}...(1+x_r)^{-1/2}$$

$$\times\frac{1}{(2\pi\omega)^r}\int_{L_1}...\int_{L_r}\left(\phi_1(s_1,...,s_r)\,\overline{\theta}_1^{*}(s_1)...\,\overline{\theta}_r^{*}(s_r)\ z_1^{s_1}...z_r^{s_r}\,x_1^{\sigma_1s_1}...x_r^{\sigma_rs_r}\right)ds_1...ds_r\Bigg]dx_1...dx_r, \quad (2.9)$$

where $\phi_1(s_1,...,s_r)$ and $\overline{\theta}_i^{*}(s_i)$, $i=1,...,r$ are given by (1.11) and (1.12) respectively.

Interchanging $s_1,...,s_r$ integrals with $x_1,...,x_r$ integrals, (2.9) becomes:

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}...\int_{L_r}\phi_1(s_1,...,s_r)\,\overline{\theta}_1^{*}(s_1)...\,\overline{\theta}_r^{*}(s_r)\ z_1^{s_1}...z_r^{s_r}\int_0^\infty...\int_0^\infty\Bigg[x_1^{\rho_1-1}...x_r^{\rho_r-1}\left(1+\sqrt{1+x_1}\right)^{v_1}...\left(1+\sqrt{1+x_r}\right)^{v_r}$$

$$\times(1+x_1)^{-1/2}...(1+x_r)^{-1/2}\ x_1^{\sigma_1s_1}...x_r^{\sigma_rs_r}\ dx_1...dx_r\Bigg]ds_1...ds_r \quad (2.10)$$

Using (1.15), (2.10) becomes:

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\left[\phi_1(s_1,\ldots,s_r)\overline{\theta}_1^{*}(s_1)\cdots\overline{\theta}_r^{*}(s_r)\,z_1^{s_1}\cdots z_r^{s_r}\right.$$

$$\left.\times\frac{1}{\left(\sqrt{\pi}\right)^r}\prod_{i=1}^{r}\Gamma\begin{bmatrix}\rho_i+\sigma_i s_i, & 1-\frac{\nu_i}{2}-\rho_i-\sigma_i s_i, & \frac{1-\nu_i}{2}-\rho_i-\sigma_i s_i\\ & 1-\nu_i-\rho_i-\sigma_i s_i & \end{bmatrix}\right] ds_1\ldots ds_r \qquad (2.11)$$

Now interpret this integral with the help of (1.8) to get the right-hand side of (2.1).

Corollary 2.2: When $r=2$, (2.1) reduces to:

$$\mathrm{M}\left[\left(1+\sqrt{1+x_1}\right)^{\nu_1}\left(1+\sqrt{1+x_2}\right)^{\nu_2}(1+x_1)^{-1/2}(1+x_2)^{-1/2}\,\overline{I}_1\left[z_1x_1^{\sigma_1},z_2x_2^{\sigma_2}\right]\right]$$

$$=\frac{1}{\sqrt{\pi}^r}\,\overline{I}_{1\;P,Q:\;p_1+2,q_1+2\;;\,p_2+2,q_2+2}^{0,0:\;m_1+2,n_1+1\;;\,m_2+2,n_2+1}\begin{bmatrix}z_1 & J_1\\ z_2 & J_2\end{bmatrix} \qquad (2.12)$$

where

$$J_1={}_1\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)};A_j\right)_P:\left(1-\rho_1,\sigma_1;1\right),{}_1\left(c_j^{(1)},\gamma_j^{(1)};1\right)_{n_1},{}_{n_1+1}\left(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)}\right)_{p_1},\left(1-\nu_1-\rho_1,\sigma_1;1\right);$$
$$\left(1-\rho_2,\sigma_2;1\right),{}_1\left(c_j^{(2)},\gamma_j^{(2)};1\right)_{n_2},{}_{n_2+1}\left(c_j^{(2)},\gamma_j^{(2)};C_j^{(2)}\right)_{p_2},\left(1-\nu_2-\rho_2,\sigma_2;1\right) \qquad (2.13)$$

$$J_2={}_1\left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)};B_j\right)_Q:\left(1-\frac{\nu_1}{2}-\rho_1,\sigma_1;1\right),\left(\frac{1-\nu_1}{2}-\rho_1,\sigma_1;1\right),{}_1\left(d_j^{(1)},\delta_j^{(1)};1\right)_{m_1},{}_{m_1+1}\left(d_j^{(1)},\delta_j^{(1)};D_j^{(1)}\right)_{q_1};$$

$$\left(1-\frac{\nu_2}{2}-\rho_2,\sigma_2;1\right),\left(\frac{1-\nu_2}{2}-\rho_2,\sigma_2;1\right),{}_1\left(d_j^{(2)},\delta_j^{(2)};1\right)_{m_2},{}_{m_2+1}\left(d_j^{(2)},\delta_j^{(2)};D_j^{(2)}\right)_{q_2}, \qquad (2.14)$$

provided the conditions are similar to that of (2.1) with $r=2$.

Corollary 2.3: When $r=1$ and $P=Q=0$, (2.1) reduces to the Mellin transform involving I - function of one variable.

Corollary 2.4: If all the exponents $A_j\,(j=1,\ldots,P)$, $B_j\,(j=1,\ldots,Q)$, $C_j^{(i)}\,(j=n_i+1,\ldots,p_i;\,i=1,\ldots,r)$, $D_j^{(i)}\,(j=m_i+1,\ldots,q_i;\,i=1,\ldots,r)$ in (2.1) are equal to unity, then it reduces to the corresponding Mellin-transform involving H - function of r variables.

Corollary 2.5: If all the exponents $A_j\,(j=1,\ldots,P)$, $B_j\,(j=1,\ldots,Q)$, $C_j^{(i)}\,(j=n_i+1,\ldots,p_i;\,i=1,\ldots,r)$, $D_j^{(i)}\,(j=m_i+1,\ldots,q_i;\,i=1,\ldots,r)$ in (2.1) are equal to unity, and $r=1$, then it reduces to the corresponding Mellin-transform involving H - function of one variable.

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