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Fuzzy Lattice KS-Operator Group

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Abstract:
In this paper a fuzzy set is defined on a group with two operators which is also a lattice satisfying four conditions. The operator sets are denoted by K and S which are any nonempty sets. The first two conditions are according to group structure and the last two conditions are according to lattice structure.

Keywords: Lattice group, Fuzzy lattice group, Fuzzy lattice KS operator group.

Introduction-
A fuzzy algebra has become an important branch of research. A. Rosenfeld 1971 [9] used the concept of fuzzy set theory due to Zadeh 1965 [5]. Since then the study of fuzzy algebraic substructures are important when viewed from a Lattice theoretic point of view. N. Ajmal and K.V. Thomas [1] initiated such types of study in the year 1994. It was latter independently established by N. Ajmal [1] that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all fuzzy sub groups of a given group and is Modular. Nanda[8] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in details and in the lattice theoretical aspects of fuzzy sub groups and fuzzy normal sub groups are explored. G.S.V. Satya Saibaba [3] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l- groups. J.A. Goguen [4] replaced the valuation set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A Solairaju and R. Nagarajan [11] introduced the concept of lattice valued Q-fuzzy sub-modules over near rings with respect to T-norms. DrM.Marudai & V. Rajendran[6] modified the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. Gu [12] introduced concept of fuzzy groups with operator. Then S. Subramanian, R Nagrajan & Chellappa [10] extended the concept to m fuzzy groups with operator. In this paper we introduce the notion of fuzzy lattice o KS operator group and investigated some of its basic properties.

1. PRELIMINARIES

Definition 1.1 Fuzzy group

Let λ: X to [0, 1] is a fuzzy set & (G,.) is a group which is a subset of X. A fuzzy group is a fuzzy set which satisfy two conditions

1) $\lambda\,(x\,y) \geq \text{mini}\,\{\lambda\,(x), \lambda(y)\}$
2) $\lambda\,(x^{-1}) \geq \lambda\,(x)$ where x, y∈ G .

Definition 1.2 K-Operator group

A group G is said to be an K- operator group if kx ∈ G where k∈ K (any non empty set called as Operator set) and for all x∈ G.

Definition 1.3 Fuzzy K- operator group

Let λ: X to [0, 1] is a fuzzy set & G is a subset of X which is also a K- operator group. λ is a fuzzy K-operator group if it satisfy following two conditions

i) $\lambda\,(k(xy)) \geq \text{mini}\,\{\lambda(kx), \lambda\,(ky)\}$
ii) $\lambda\,(kx)^{-1} \geq \lambda(kx)$ where x, y∈ G, k ∈ K.

Definition 1.4 Lattice K-operator group

Lattice K-operator group is an algebraic structure (G, . , R) if it satisfy two conditions 1) G is a K-operator group w.r.t '.' 2) G is a lattice w.r.t R

Definition 1.5 KS- operator group-

Let G be a group, K,S be any twononempty sets if kx ϵ G , sxϵ G. for every x ϵ G, k ϵ K, sϵ S Then G is called a KS- operator group.

Definition 1.6 Fuzzy KS- operator group

If λ: X to [0, 1] is a fuzzy set & G is KS- operator group . A fuzzy set λ over G, G subset of X is a fuzzy KS operator group if

1) λ (kxsy)) ≥ mini {λ (kx), λ (sy)} 2) λ (kx)$^{-1}$ ≥ λ (kx) &λ (sx)$^{-1}$ ≥ λ (sx) for every x, yϵ G, k ϵ K ,s ϵ S

Definition 1.7 Lattice KS operator group

A lattice KS- operator group is an algebraic structure (G, R , .) if it satisfy two conditions 1) G is a KS-operator group w.r.t '.' 2) G is a lattice w.r.t R .

Definition 1.8 Fuzzy lattice KS- operator group (FL KS- operator group) –

λ: X to [0, 1] be a fuzzy set, Let G be a subset of X which is a lattice KS- operator group , K,S(operator sets). λ is a function over G. It is a fuzzy lattice KS- operator group if it satisfy following four conditions

1) λ(kxsy)≥mini { λ (kx),λ (sy)}
2) λ(kx)$^{-1}$ ≥λ(kx) &λ(sx)$^{-1}$≥λ(sx)
3) λ(kx∨sy)≥mini{λ(kx),λ(sy)}
4) λ(kx∧sy)≥mini{λ(kx),λ(sy)} For every x ϵ G, k ϵ K, s ϵ S

Definition 1.9 Fuzzy lattice KK -operator group

λ: X to [0, 1] is a fuzzy set; G is a K- lattice operator group, A function λ on G is said to be a fuzzy lattice KK-operator group if it satisfy following four conditions

1) λ($k_1x\ k_2y$)≥ mini{λ($k_1\ x$),λ($k_2\ y$)}
2) λ($k_1\ x$)$^{-1}$≥λ(k_1x), λ($k_2\ x$)$^{-1}$≥λ(k_2x),
3) λ(k_1x ∨ k_2y)≥ mini{λ($k_1\ x$),λ($k_2\ y$)}
4) λ(k_1x∧ k_2y)≥ mini{λ($k_1\ x$),λ($k_2\ y$)}, For all x, yϵ G , k_1,k_2ϵK

Definition 1.10 Fuzzy lattice K^2-operator group

λ: X to [0, 1] is afuzzy set; G is a K- lattice operator group, A function λ on G is said to be a fuzzy lattice K-operator group if it satisfy following four conditions

1) λ($kxky$)≥mini{λ(kx),λ(ky)}
2) λ(kx) $^{-1}$≥λ(kx)
3) λ(kx∨ky) ≥mini{λ(kx),λ(ky)}
4) λ(kx∧ky) ≥mini{λ(kx),λ(ky)} For all x, y ϵ G , kϵ K

Definition 1.11 Let λ : X to Y be a function. Q is a fuzzy group of Y. A fuzzy set λ^{-1} Inverse image of Q under λ is given by λ^{-1}(Q) = $\mu_{\lambda^{-1}(Q)}$ (x) = $\mu_Q\lambda$ (x)

Definition 1.12 μ_A: X to [0, 1] be a fuzzy set and λ: X to X' is a function. A functionμ_{A^λ} : X to [0,1] is defined by μ_{A^λ} (x) = μ_A λ (x)

Definition 1.13 If T and T' are lattice KS- operator groups . A function λ: T→T' be a lattice KS homomorphism if λ(kxsy)=λ(kx)λ(sy)=kλ(x)sλ(y),λ(kx∨sy)=λ(kx)∨λ(sy)=kλ(x)∨sλ(y), λ(kx∧sy)= λ(kx)∧λ(sy) = kλ(x)∧sλ(y) For all x , y ϵ G, k ϵ K, s ϵ S

Definition 1.14 Let A_i be a fuzzy lattice KS operator group of G_i, for i = 1,2,……,n. Then the product A_i(i = 1,2,…..n) is the function A_1x A $_2$x …….. x A_n : G_1x G $_2$x …….. x G $_n$ → [0,1] defined by(A_1x A $_2$x …….. x A_n) k (x $_1$,x $_{2,\ldots\ldots}$,x $_n$) = mini{A_1 (k x $_1$),A_2(kx_2),…….,A_n(kx_n)}

2 PROPERTIES OF FL KS- OPERATOR GROUP

Preposition 2.1: Let T and T' be two Lattice KS-operator groups andλ: T to T' be a lattice KS homomorphism. If P' is a FL KS operator group of T' then the pre-image

$\lambda^{-1}(P')$ is a FL KS operator group of T.

Proof- Assume P' is a FL KS- operator group of T'. Let x, y ϵ T

i) $\lambda^{-1}(P') = \mu_{\lambda^{-1}(P')}(kxsy)) = \mu_{P'},\lambda(kxsy)) = \mu_{P'}(k\lambda(x)s\lambda(y))$
$\geq \text{mini}\{\mu_{P'}(k\lambda(x)),\mu_{P'}(s\lambda(y))\} \geq \text{mini}\{\mu_{P'}(\lambda(kx)),\mu_{P'}(\lambda(sy))\}$
$= \text{mini}\{\mu_{\lambda^{-1}(P')}(kx),\mu_{\lambda^{-1}(P')}(sy)\}$

ii) $\mu_{\lambda^{-1}(P')}(kx)^{-1} = \mu_{P'}\lambda(kx)^{-1} = \mu_{P'}[\lambda(kx)]^{-1} = \mu_{P'}[k\lambda(x)]^{-1} \geq \mu_{P'}\ k\lambda(x)$
$= \mu_{P'}(\lambda(kx)) = \mu_{\lambda^{-1}(P')}(kx)\mu_{\lambda^{-1}(P')}(sx)^{-1} = \mu_{P'}\lambda(sx)^{-1} = \mu_{P'}[\lambda(sx)]^{-1}$
$= \mu_{P'}[s\lambda(x)]^{-1} \geq \mu_{P'}s\lambda(x) = \mu_{P'}(\lambda(sx)) = \mu_{\lambda^{-1}(P')}(sx)$

iii) $\mu_{\lambda^{-1}(P')}(kx \vee sy) = \mu_{P'}\lambda(kx \vee sy) = \mu_{P'}\lambda(kx) \vee \lambda(sy) \geq \text{mini}\{\mu_{P'}\lambda(kx),\mu_{P'}\lambda(sy))$
$\geq \text{mini}\{\mu_{\lambda^{-1}(P')}(kx),\mu_{\lambda^{-1}(P')}(sy)\}$

iv) $\mu_{\lambda^{-1}(P')}(kx \wedge sy) = \mu_{P'}\lambda(kx \wedge sy) = \mu_{P'}\lambda(kx) \wedge \lambda(sy) \geq \text{mini}\{\mu_{P'}\lambda(kx),\mu_{P'}\lambda(sy))$
$\geq \text{mini}\{\mu_{\lambda^{-1}(P')}(kx),\mu_{\lambda^{-1}(P')}(sy)\}$

Therefore $\lambda^{-1}(P')$ is a FL KS- operator group of T.

Preposition 2.2: If T and T' are two Lattice KS operator groups and λ:T to T' is a lattice KS epimorphism. P' is a fuzzy set in T'. If $\lambda^{-1}(P')$ is a FL KS operators group of T then P' is a FL KS- operators group of T'.

Proof- Consider x, y ϵ T', hence there is are elements m, n ϵT so that λ(m)= x and λ(n)= y.

i) $\mu_{P'}(kxsy) = \mu_{P'}(k\lambda(m)s\lambda(n)) = \mu_{P'}(\lambda(kmsn))$
$= \mu_{\lambda^{-1}(P')}(kmsn) \geq \text{mini}\{\mu_{\lambda^{-1}(P')}(km)),\mu_{\lambda^{-1}}(P')(sn)\}$
$\geq \text{mini}\{\mu_{P'}\lambda(km),\mu_{P'}\lambda(sn)\} \geq \text{mini}\{\mu_{P'}k\lambda(m),\mu_{P'}s\lambda(n)\}$
$\geq \text{mini}\{\mu_{P'}(kx),\mu_{P'}(sy)\}$

ii) $\mu_{P'}(kx)^{-1} = \mu_{P'}(k\lambda(m))^{-1} = \mu_{P'}(\lambda(km)^{-1} = \mu_{\lambda^{-1}(P')}(km)^{-1} \geq \mu_{\lambda^{-1}(P')}(km)$
$= \mu_{P'}\lambda(km) = \mu_{P'}k\lambda(m) = \mu_{P'}(kx)$
$\mu_{P'}(sx)^{-1} = \mu_{P'}(s\lambda(m))^{-1} = \mu_{P'}(\lambda(sm)^{-1} = \mu_{\lambda^{-1}(P')}(sm)^{-1} \geq \mu_{\lambda^{-1}(P')}(sm) = \mu_{P'}\lambda(sm)$
$= \mu_{P'}s\lambda(m) = \mu_{P'}(sx)$

iii) $\mu_{P'}(kx \vee sy) = \mu_{P'}(k\lambda(m) \vee s\lambda(n)) = \mu_{P'}(\lambda(km) \vee \lambda(sn))$
$= \mu_{P'}(\lambda(km \vee sn)) = \mu_{\lambda^{-1}(P')}(km \vee sn) \geq \text{mini}\{\mu_{\lambda^{-1}(P')}(km),\mu_{\lambda^{-1}(P')}(sn)\}$
$= \text{mini}\{\mu_{P'}\lambda(km),\mu_{P'}\lambda(sn)\} = \text{mini}\{\mu_{P'}k\lambda(m),\mu_{P'}s\lambda(n)\}$
$= \text{mini}\{\mu_{P'}(kx),\mu_{P'}(sy)\}$

iv) $\mu_{P'}(kx \wedge by) = \mu_{P'}(k\lambda(m) \wedge s\lambda(n)) = \mu_{P'}(\lambda(km) \wedge \lambda(sn))$
$= \mu_{P'}(\lambda(km \wedge sn)) = \mu_{\lambda^{-1}(P')}(km \wedge sn) \geq \text{mini}\{\mu_{\lambda^{-1}(P')}(km),\mu_{\lambda^{-1}}(P')(sn)\}$
$= \text{mini}\{\mu_{P'}\lambda(km),\mu_{P'}\lambda(sn)\} = \text{mini}\{\mu_{P'}k\lambda(m),\mu_{P'}s\lambda(n)) = \text{mini}\{\mu_{P'}(kx),\mu_{P'}(by)\}$

Therefore P' is a FL KS operator group of T'.

Preposition 2.3: If $\{A_i\}$ is a family of FL KS operator group of T then $\cap A_i$ is a FL KS operator group of T where $\cap A_i = \{ x , \wedge\lambda_{Ai}(x) / x \in T \}$

Proof- Consider x, y ϵ T

i) $(\cap \lambda_{Ai})(kxsy) = \wedge\lambda_{Ai}(kxsy) \geq \wedge\ \text{mini}\{\lambda_{Ai}(kx),\lambda_{Ai}(sy)\}$
$= \text{mini}\{(\cap \lambda_{Ai})(kx),(\cap \lambda_{Ai})(sy)\}$

ii) $(\cap \lambda_{Ai})(kx)^{-1} = \wedge\lambda_{Ai}(kx)^{-1} \geq \wedge\lambda_{Ai}(kx) = (\cap \lambda_{Ai})(kx)$
$(\cap \lambda_{Ai})(sx)^{-1} = \wedge\lambda_{Ai}(sx)^{-1} \geq \wedge\lambda_{Ai}(sx) = (\cap \lambda_{Ai})(sx)$

iii) $(\cap \lambda_{Ai})(kx \vee sy) = \wedge\lambda_{Ai}(kx \vee sy) \geq \wedge\ \text{mini}\{\lambda_{Ai}(kx),\lambda_{Ai}(sy)\} = \text{mini}\{(\cap \lambda_{Ai})(kx),(\cap \lambda_{Ai})(sy)\}$

iv) $(\cap \lambda_{Ai})(kx \wedge sy) = \wedge\lambda_{Ai}(kx \wedge sy)$
$\geq \wedge\ \text{mini}\{\lambda_{Ai}(kx),\lambda_{Ai}(sy)\} \geq \text{mini}\{(\cap \lambda_{Ai})(kx),(\cap \lambda_{Ai})(sy)\}$

Therefore $\cap A_i$ is a FL KS operator group of T

Preposition 2.4: P is a FL KS operator group of T. If Q is a fuzzy set in T given by Q(x)=P(x)–P(e)+1 for every x ϵT. Then Q is a FL KS operator group of T consisting P.

Proof – Consider x, y ϵ T

i) $Q(kxsy)$ =P(kxsy)+1-P(e) ≥mini({P(kx),P(sy)}+1–P(e)) ≥mini(P(kx)+1-P(e),P(sy)+1–P(e))≥mini{Q(kx),Q(sy)}
ii) Q((kx)$^{-1}$) = P((kx)$^{-1}$)+1–P(e) ≥P(kx)+1–P(e) ≥Q(kx)
Q((sx)$^{-1}$) =P((sx)$^{-1}$)+1–P(e) ≥P(sx)+1–P(e)≥Q(sx)
iii) Q(kx∨sy) =P(kx∨sy)+1-P(e) ≥mini{P(kx),P(sy)}+1–P(e) ≥mini{P(kx)+1-P(e),P(sy)+1–P(e)}≥mini{Q(kx),Q(sy)}
iv) Q(kx∧sy) =P(kx∧sy)+1-P(e) ≥mini{P(kx),P(sy)}+1–P(e) ≥mini{P(kx)+1-P(e),P(sy)+1–P(e)}≥mini{Q(kx),Q(sy)}
Also P(x)≤Q(x) for all x ϵ T.
Therefore Q is a FL KS operator group of T consisting P.

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